Connections of Pair Correlations to Equidistribution and Additive Energy

Let $(x_n)_n$ be an arbitrary sequence of real numbers. Let $\{x\}$ denote the fractional part of x and $\|x\| = \mathrm{d}(x, \mathbb{Z})$, then $(x_n)_n$ is said to be equidistributed modulo 1 if

$$\lim_{N\to\infty} \frac{1}{N} \# \{1 \leq n \leq N : \{x_n\} \in [a,b]\} = b - a$$

for all $0 \leq a \leq b \leq 1$, and is said to have Poissonian pair correlations if

$$\lim_{N\to\infty} \frac{1}{N} \# \left\{1 \leq m \neq n \leq N : \|x_m - x_n\| \leq \frac{s}{N}\right\} = 2s$$

for all $s \geq 0$.

While it is widely studied under which conditions a given sequence is equidistributed modulo 1, there was not much known so far for sequences to have Poissonian pair correlations.

In this talk we will see that, in fact, to have Poissonian pair correlations is a stronger property for a sequence than to be equidistributed. (see arXiv:1612.05495 for more information on this)

Furthermore, we will deepen the investigations for sequences $(\alpha a_n)_n$, $\alpha \in \mathbb{R}$ and $a_n \in \mathbb{N}$ increasing, how the additive Energy $E(A_N)$ of $A_N = \{a_n\}_{n \leq N}$ influences the property of $(\alpha a_n)_n$ to have Poissonian pair correlations where

$$E(A_N) = \sum_{\substack{a,b,c,d \in A_N \\ a+b=c+d}} 1.$$

(see arXiv:1606.03591 for more information on this)