



The Frontiers of Superstring Theory :

"D-branes" and new perspective of our world



Koji Hashimoto (橋本幸士)
Theoretical Physics Labo (川合理論研)

Braneworld

“Brane”

← “membrane”



Extra dimensions

Sec.1

Why does this new picture arise?

Branes predicted by Superstring theory
are expected to solve gravity problems
in elementary particle theory

5 slides

Sec.2

Can we observe them?

5 slides

Sec.3

Are they useful
for other theories and sciences?

5 slides

0. Need of Superstring Theory?

Road to the unification of all forces and matter

Present basis of physics

Quantum Mechanics

Heisenberg, Schroedinger (1925)

Standard model of
elementary particle physics,
Nuclear physics, Chemistry,
Condensed matter physics,..

Gravity

Einstein (1914)

Big Bang Cosmology
Inflation

Unify

Unified Theory : for all forces and matter

Need of Superstring theory

Naïve quantum treatment of Einstein's gravity

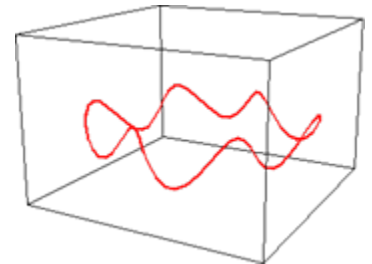


Divergence in physical quantities!

(Theory is ill-defined)

Superstring Theory : no divergence!

Quantum theory including gravity,
but, not a theory of particles!!!



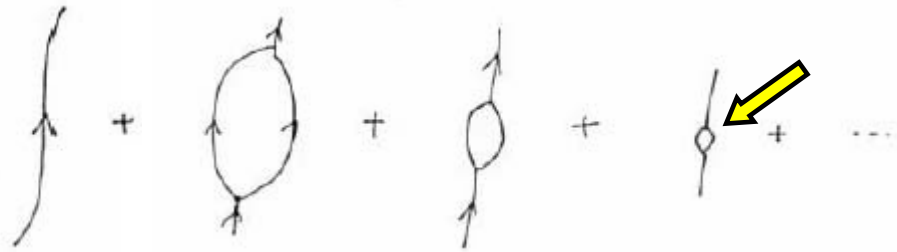
= Candidate for the unified theory

We expect all the forces and matters are derived
from the superstring theory

Strings cure the problem

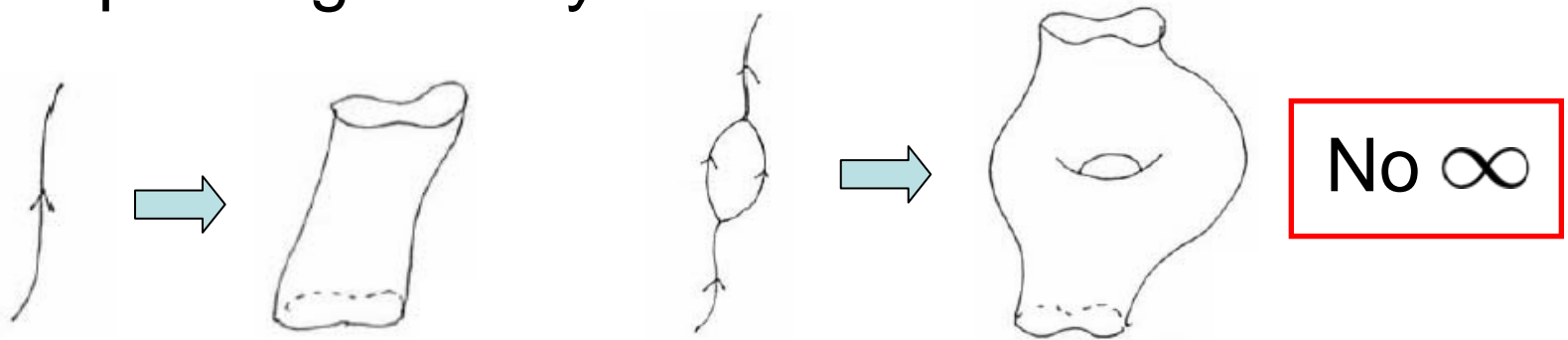
Feynman's way of Quantum mechanics: Path integral

One needs to sum up all possible paths



Small loop gives the divergence....

In Superstring Theory ...



Interaction points are smeared

Two immediate questions

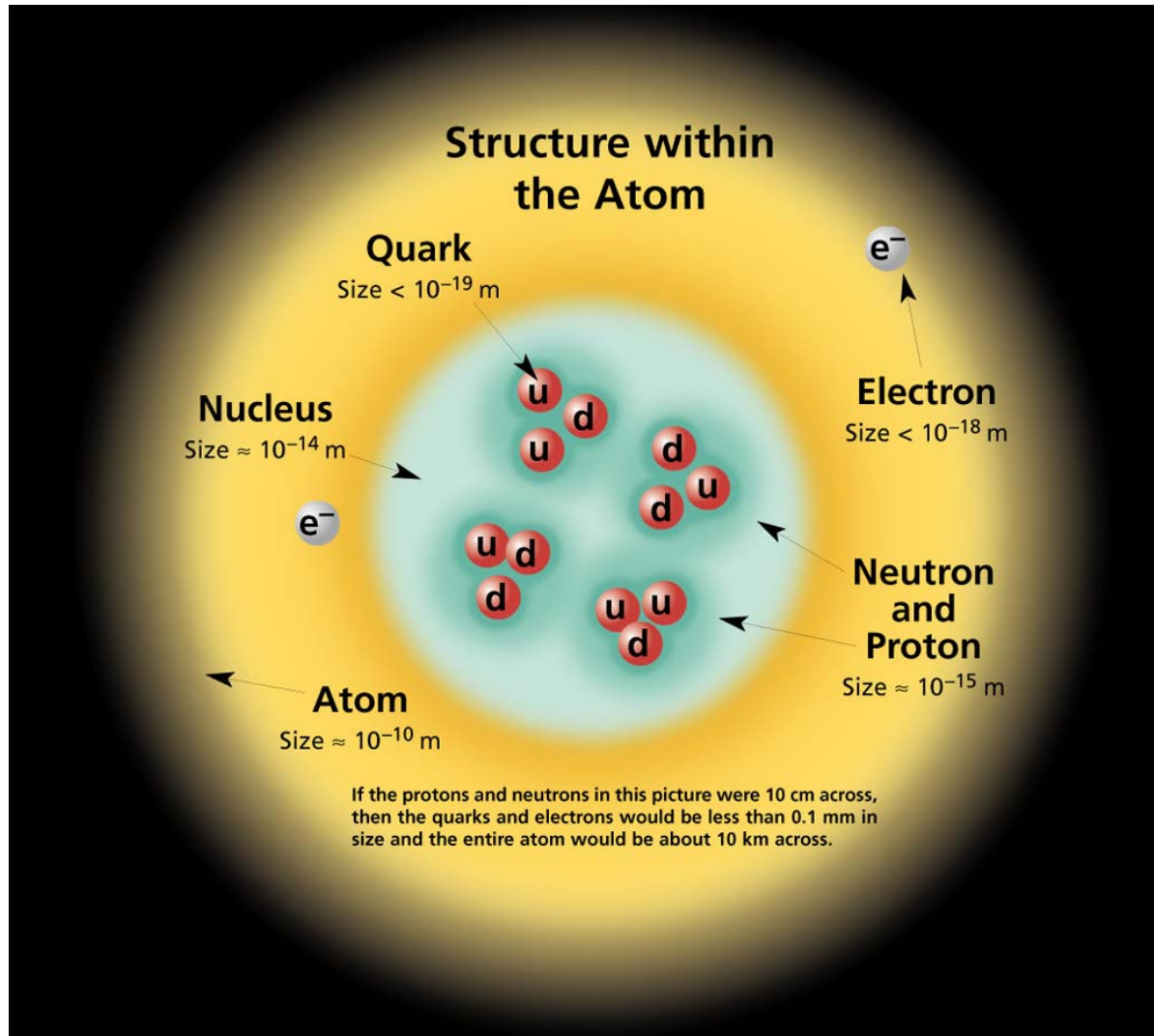
Superstrings have not been observed ... why?

In high energy experiments, the effect of quantum gravity is negligibly small

Prediction of superstring theory?

Number of Spatial dimensions is not 3

Sub-atomic structure



Standard model of elementary particles

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.003	2/3
d down	0.006	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	175	2/3
b bottom	4.3	-1/3

Leptons spin = 1/2		
Flavor	Mass GeV/c ²	Electric charge
ν_e electron neutrino	$<1 \times 10^{-8}$	0
e electron	0.000511	-1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	-1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	-1

Unified Electroweak spin = 1		
Name	Mass GeV/c ²	Electric charge
γ photon	0	0
W⁻	80.4	-1
W⁺	80.4	+1
Z⁰	91.187	0
Strong (color) spin = 1		
Name	Mass GeV/c ²	Electric charge
g gluon	0	0

Standard Model (SM)

- Constructed in 1970's
- Describes multi-particles in quantum manner.
- Gives interactions between particles
- Explains almost all particle physics experiments

Why is it okay without gravity?

Gravitational force is negligibly small

Newton's law $F = G \frac{m_1 m_2}{r^2}$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$$

Planck mass :

$$M_{pl} = \frac{1}{\sqrt{G}} = 10^{19} \text{ GeV}$$

in the natural unit $c = 1, h/2\pi = 1$

Compare it with top quark mass

$$M_t \sim 175 \text{ GeV} \quad : \text{small !!!}$$



Why is it okay without gravity?



Gravitational force is negligibly small

Effect of Quantum Gravity
is so small

Superstring effect is so small

String is difficult to observe....

However, quantum theory
of gravity needs superstrings!

New Question :
Why is it so small?

Branes may answer this!

Theoretical consequence of Superstring Theory

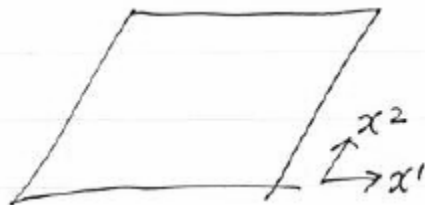
Prediction : 10 Spacetime dimensions

Superstring + Relativity \rightarrow 10 dimensions

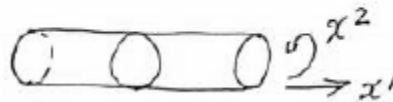
Then where are the extra dimensions?!

Solution

Hypothesis : Compactified space

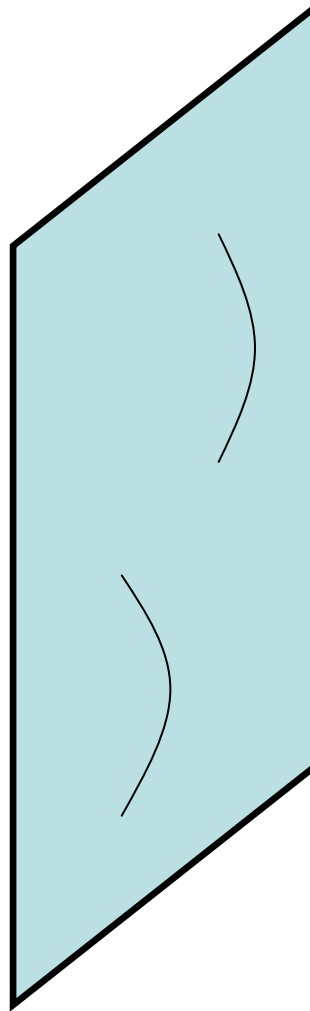


2 dim



Effectively 1 dim

Another Prediction



1. Braneworld



Why Strings?

A naïve question : Why strings, not membranes?

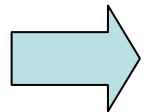
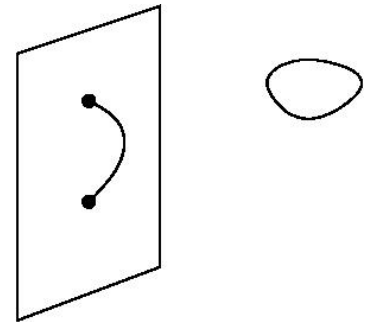
Answer: Both, at the same time!

Superstring theory inevitably introduces “branes”

**D-branes = Hypersurface on which
superstrings can end**

“D” ← Dirichlet boundary condition

“brane” ← membrane



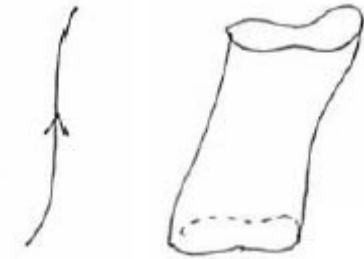
Theory of superstrings and branes

||

unified theory

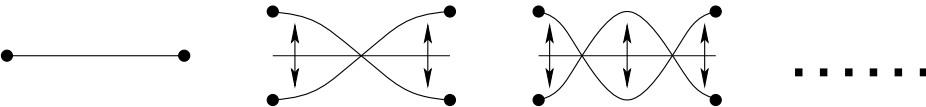
Particles from superstrings

Description of a string : $X^\mu(t, \sigma)$
($\mu = 1, 2, \dots, D - 1$)



Cf) Particle : $X^\mu(t)$

Equation of motion of a string : $\frac{\partial^2}{\partial t^2} X^\mu - \frac{\partial^2}{\partial \sigma^2} X^\mu = 0$

Solution : 

Each fluctuation looks as a particle !

→ Infinite kinds of particles = a string

Fluctuation direction : $\mu = 1, 2, \dots, D - 1$

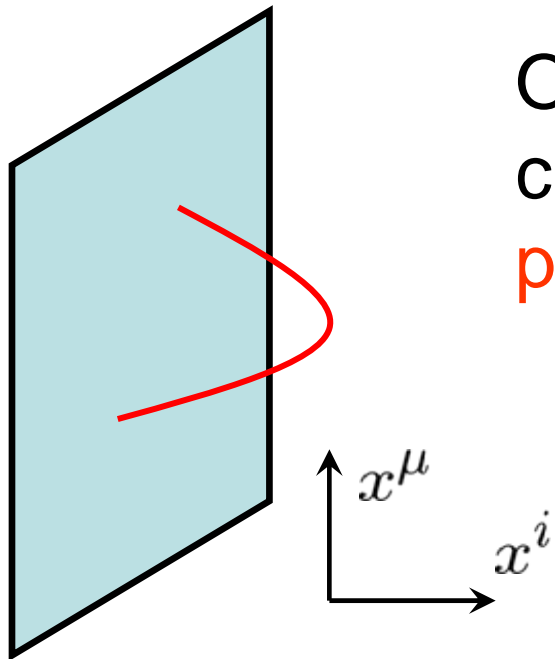
→ Photon (elemag wave) polarization $A_\mu(x)$

D-branes and photons

Boundary conditions for open superstrings :

$$\text{Neumann : } \frac{\partial}{\partial \sigma} X^\mu(\sigma = 0) = 0 \quad (\mu = 0, 1, \dots, p)$$

$$\text{Dirichlet : } X^i(\sigma = 0) = c \quad (i = p + 1, \dots, D - 1)$$



Open superstrings (**photons**)
can propagate only on the
 $p+1$ dimensional brane

“D **p** -branes”

\neq pea brains

Gravity from closed superstrings

Gravity describes curved geometry

Gravitational field : $g_{\mu\nu}(x)$

Quanta = “gravitons”

Closed superstring has two oscillations :

$$\frac{\partial^2}{\partial t^2} X^\mu - \frac{\partial^2}{\partial \sigma^2} X^\mu = 0, \quad X^\mu(\sigma + 2\pi) = X^\mu(\sigma)$$

$$\rightarrow \text{sol : } X^\mu(t, \sigma) = f^\mu(t + \sigma) + g^\mu(t - \sigma)$$

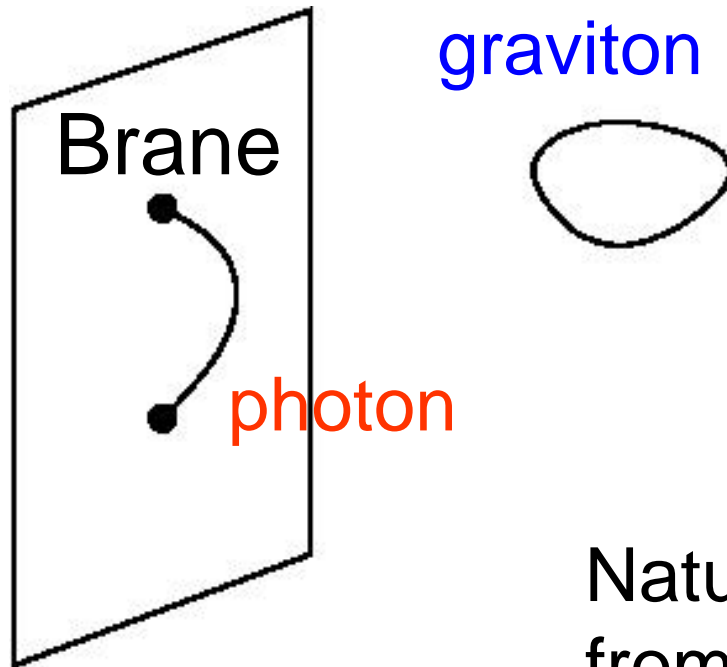
Left- and right-movers fluctuate

\rightarrow two-index particle \rightarrow graviton field $g_{\mu\nu}(x)$

Brane World

Open superstrings (**photons**) : in the brane

Closed superstrings (**gravitons**) : can be away



**We are living
in the branes!**

Natural theoretical output
from superstring theory

2. Can we observe the braneworld?



Braneworld is consistent?!

Newton's law for Gravity :

$$F = \frac{1}{M_{pl}^2} \frac{m_1 m_2}{r^2} \iff V(r) = -\frac{1}{M_{pl}^2} \frac{m_1 m_2}{r}$$

In $D(> 4)$ dimensions,

modified

$$V(r) = -\frac{1}{M_{pl(D)}^{D-2}} \frac{m_1 m_2}{r^{D-3}}$$

Of course, this contradicts with observation in solar system

a way out : Compactified extra dimensions

Assume that extra dimensions are compactified by a circle with radius R 

$$\rightarrow \left\{ \begin{array}{l} V(r) = -\frac{1}{M_{pl(D)}^{D-2}} \frac{m_1 m_2}{r^{D-3}} \quad r \ll R \\ V(r) = -\frac{1}{M_{pl(D)}^{D-2}} \frac{1}{R^{D-4}} \frac{m_1 m_2}{r} \quad r \gg R \end{array} \right.$$

If the latter is our familiar Newton's law,

$$M_{pl}^2 = M_{pl(D)}^{D-2} R^{D-4} = 10^{38} \text{ (GeV)}^2$$

Consistent, and

Resolution of the smallness problem

$$M_{pl}^2 = M_{pl(D)}^{D-2} R^{D-4} = 10^{38} \text{ (GeV)}^2$$

Use this for the smallness problem!

A solution : $M_{pl(D)} \sim 10^3 \text{ GeV}$ Arkani-Hamed, Dimopoulos, Dvali

→ **Prediction** : $R \sim 10^{30/(D-4)-19} \text{ [m]}$

$D = 5 \rightarrow R \sim 10^{11} \text{ [m]}$ apparently rejected

$D = 6 \rightarrow R \sim 10^{-3} \text{ [m]}$... **Possible!!!**

In fact, Newton's law has been confirmed only down to a sub-mm scale.

→ **Future experiments can confirm the scenario**

Experimental consequence

The energy $M_{pl(D)} \sim 10^3 \text{ GeV}$ can be reached by particle colliders!

RHIC experiment



What will happen at collisions of particles with energy $M_{pl(D)} \sim 10^3 \text{ GeV}$?

Production of Blackholes at the collider!!

Argyres, Dimopoulos, March-Russell

Creation of Blackhole : Energy concentration in a region smaller than Schwarzschild radius r_s

$$r_s = Gm = m/M_{pl}^2$$

Collider energy : $m = 10^3 \text{ [GeV]}$

$$M_{pl} = 10^{19} \text{ [GeV]} \Rightarrow r_s = 10^{-51} \text{ [m]}$$

$$M_{pl(D)} = 10^3 \text{ [GeV]} \Rightarrow r_s = 10^{-19} \text{ [m]}$$

Possible !!!

radius of nuclei $\gtrsim 10^{-15} \text{ [m]}$

3. Are branes useful for other scientific fields?



New mathematical methods born in superstring theory

Gauge/Gravity Correspondence

Gauge field theories (such as electromagnetism)
= Gravity in higher dimensions

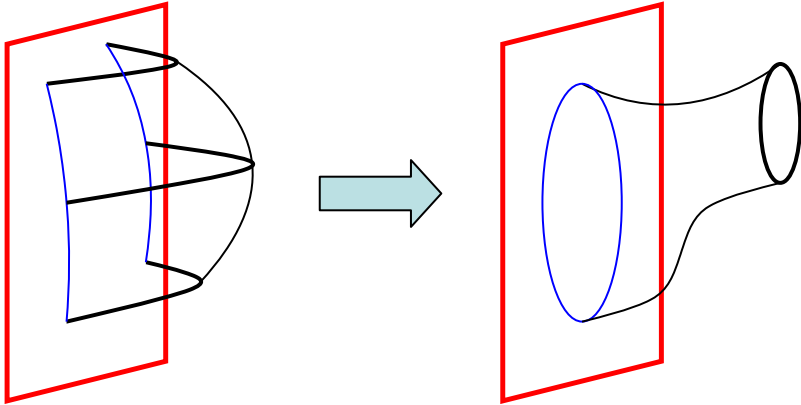
Application to → Nuclear physics

Brane-Engineering of Solitons

Solitons = Dp -branes inside Dq -branes ($p < q$)

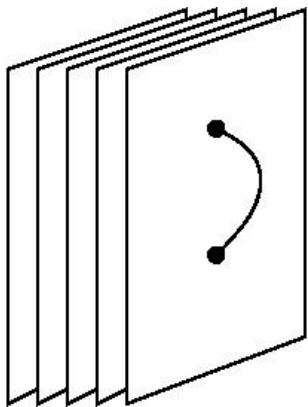
Application to → Condensed matter physics,
Cosmology, Mathematics

Another description of D-branes



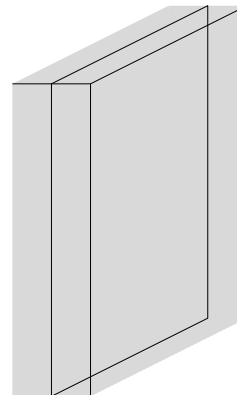
D-branes
= sources for gravity
→ blackholes

Gauge/Gravity Correspondence



Open
superstrings
in D-branes

“Gauge”



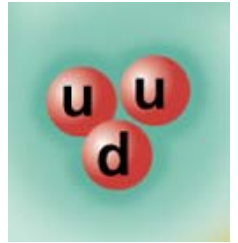
Closed
superstrings
in Blackhole
Spacetime

“Gravity”

Quarks = Gravity !

Quantum Chromodynamics (theory of quarks)

Strongly coupled difficult to analyze



Static properties of Proton / Neutron

		Gravity Analysis	Experiments
Charge Radius ²	P	$(0.8 \text{ fm})^2$	$(0.88 \text{ fm})^2$
	N	$(0.0 \text{ fm})^2$	$- 0.12 \text{ fm}^2$
Magnetic Moment	P	2.2	2.8
	N	- 1.3	- 1.9

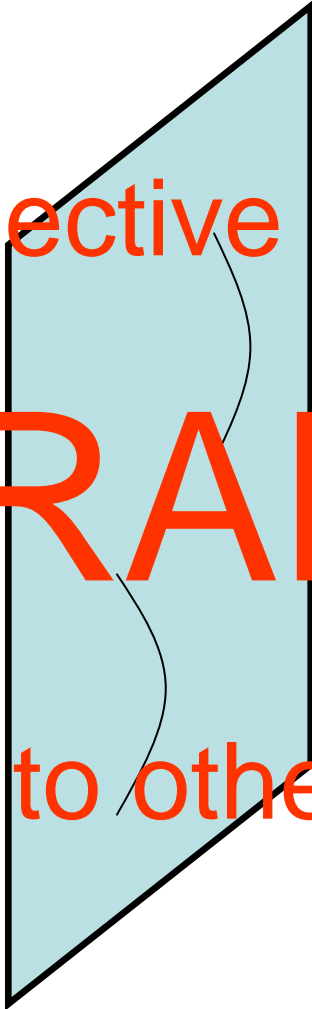
input : f_π, m_ρ

Sakai-Sugimoto-KH (2008)

Superstring theory

Quantum gravity = candidate for unified theory

New perspective of our world!



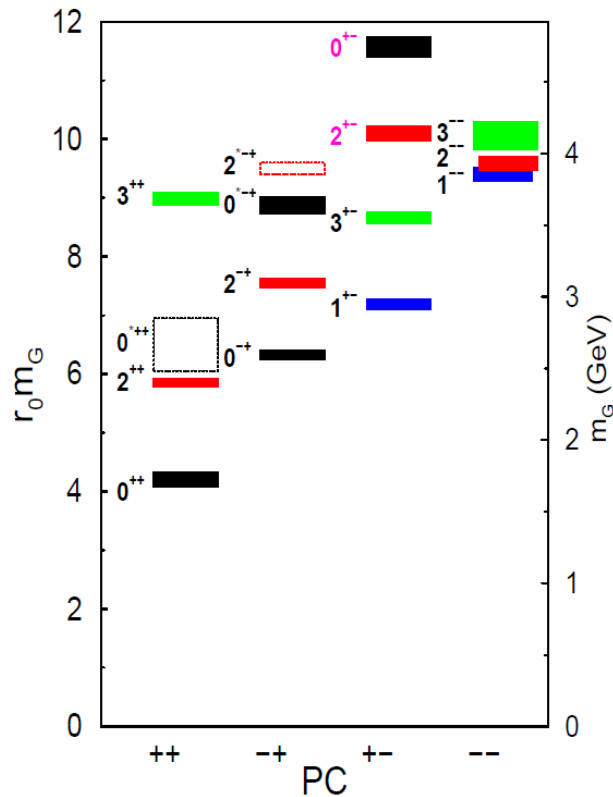
BRANE

Applications to other scientific fields!

Example of Gauge/Gravity Correspondence

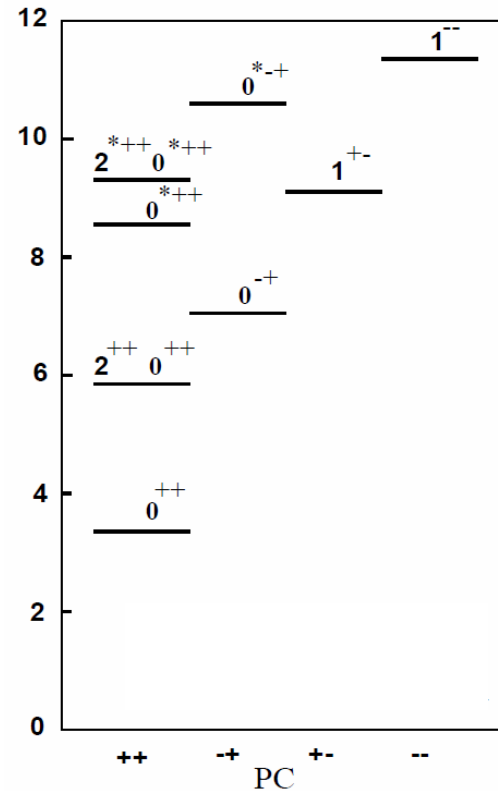
Quantum Chromodynamics (theory of quarks)

Numerical Simulation of Glueball boundstate spectra



(Morningstar/Peardon, 1999)

Gravity analysis



(Brower/Mathur/Tan, 2003)

Reconnection of Magnetic Fluxtube

D-brane theory can be used to physics of solitons

“D-brane engineering”

Superconductor



Theory in a Dq -brane

困 ↓ 難

Magnetic-flux
solution

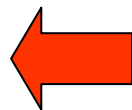


Flux = Additional Dp -brane ($p < q$)



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Study of interactions
among fluxes



Dp -brane theory

